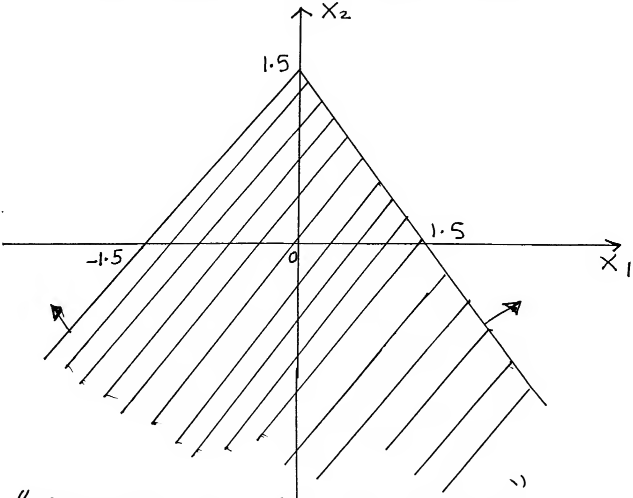
19 Consider the neural network of Fig. 3, with an input data pattern $(X_1, X_2)$ and an output signal s. All neurons of the hidden and output layers Produce binary threshold signals. Find the various weights of the network such that it behaves as a two-class data classifier with the two separation lines shown in Fig. 4. The points within the hatched region are identified by s = 0, and the points outside this region are identified by s = 1. How will the input data patterns (0.5, 0.5), (0.5, 1.5), (0, 2), and (0.9, 1.3) be classified?




" Fig. 4 Separation lines for Prob. 19 "

20 Consider the neural network of Fig. 3, with an input data pattern $(x_1, x_2)$ and an output signal $s$. All neurons of the hidden and output layers produce binary threshold signals. Find the various weights of the network such that it behaves as a two-class data classifier with the separation lines shown in Fig. 5. The points within the hatched region are identified by $s=0$, and the points outside this region are identified by $s=1$. How will the following input data patterns be classified?

(-1, 1) (-1, -1) (1, 2.4)

(-0.6, 2.7) (0.8, 0.5) (0, 1.9)

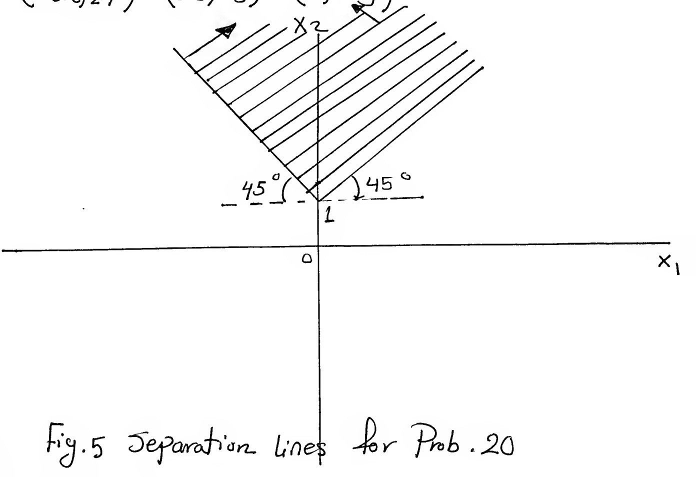



Fig. 5 Separation lines for Prob. 20

## Problem 21

Consider the neural network of Fig. 3, with an input data pattern $(x_1, x_2)$ and an output signal $s$. All neurons of the hidden and output layers produce binary threshold signals. The numerical values of the bias weights $w_{03}$ and $w_{04}$ are 1.0 and 0.8, respectively. Find the various weights of the network such that it behaves as a two-class data classifier with the separation lines shown in Fig. 6. The points within the hatched region are identified by $s = 1$, and the points outside this region are identified by $s = 0$. How will the input data patterns $(2, 0)$, $(2, 4)$, $(-4, 4)$, and $(-4, -5)$ be classified?

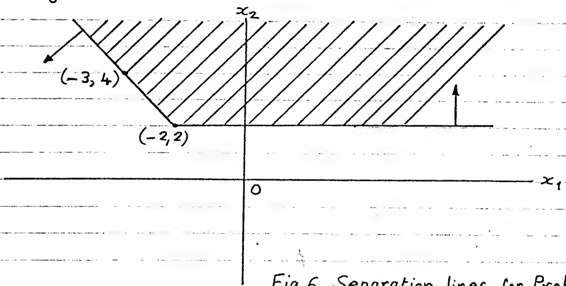



Fig. 6 Separation lines for Prob. 21

## Solution

The first separation line [passing through points $(-2, 2)$ and $(-3, 4)$] is

$$\frac{x_2 - 2}{x_1 + 2} = \frac{4 - 2}{-3 + 2}$$

22 Consider the neural network of Fig.3, with an inputs data pattern $(x_1, x_2)$ and an output signal $s$. All neurons of the hidden and output layers produce binary threshold signals. The numerical values of the bias weights $W_{03}$ and $W_{04}$ are 0.7 and 0.4, respectively. Find the various weights of the network such that it behaves as a two-class data classifier with the separation lines shown in Fig.7. The points within the hatched region are identified by $s=1$, and the points outside this region are identified by $s=0$. How will the input data patterns (0,0), (0,3), (2,2), and (2,-2) be classified?

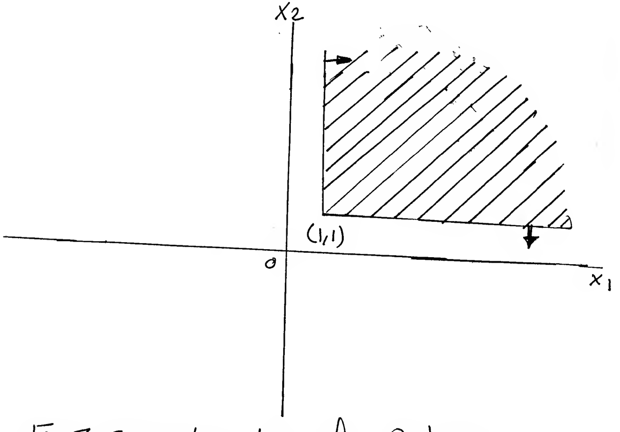



Fig.7 Separation lines for Prob.22.

## Problem 23

The neural network of Fig. 8 has an input data pattern $(x_1, x_2)$ and an output signal $s$. All neurons of the hidden and output layers produce binary threshold signals. The weight values are:

$w_{13} = -1$ , $w_{23} = 0.5$ , $w_{03} = 0.5$
$w_{14} = 0$ , $w_{24} = -0.8$ , $w_{04} = 0.8$
$w_{15} = 0.2$ , $w_{25} = 0.2$ , $w_{05} = -1$
$w_{36} = 1$ , $w_{46} = 1$ , $w_{56} = 1$
$w_{06} = -0.5$

(a) Show that the network can behave as a two-class data classifier on the $x_1 - x_2$ plane, where all points within a triangle with vertices $(1, 1)$, $(2, 3)$, and $(4, 1)$ are identified by $s = 0$, and all points outside this triangle are identified by $s = 1$.

(b) How will the input data patterns $(2, 2)$, $(2, -2)$, $(-1, 1.5)$, and $(5, 3)$ be classified?

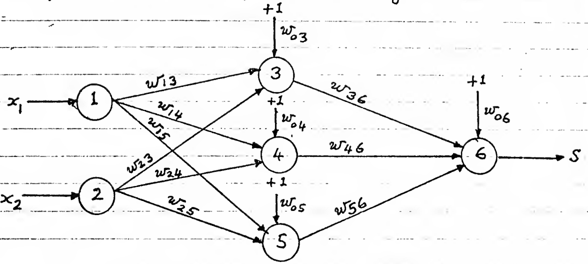



Fig. 8 Neural network for Prob. 23

24 Consider the neural network of Fig. 8, with an input data Pattern $(x_1, x_2)$ and an output signal s. All neurons of the hidden and output layers produce binary threshold signals. The weight values are:

$w_{13} = 0.3$, $w_{23} = -0.7$, $w_{03} = 0.2$

$w_{14} = 0$, $w_{24} = 1$, $w_{04} = 1$

$w_{15} = 0.25$, $w_{25} = 0$, $w_{05} = -1$

$w_{36} = 0.8$, $w_{46} = 0.7$, $w_{56} = -0.4$

$w_{06} = -1.3$

a) Show that the network can behave as a two-class data classifier on the $X_1 - X_2$ plane, where all points within a specific right-angled triangle are identified by $s = 1$, and all points outside this triangle are identified by $s = 0$.

b) Determine the vertices of the triangle referred to in Part (a).

c) What logic operation is performed by neuron 6 on the signals produced by neurons 3, 4 and 5?

d) How will the input data patterns (0, 0), (2, 0), (4.5, 0), and (-3.6, 1.7) be classified?

Problem 25.

Consider the neural network of Fig. 8, with an input data pattern $(x_1, x_2)$ and an output signal $s$. All neurons of the hidden and output layers produce binary threshold signals. Find the various weights of the network such that it behaves as a two-class data classifier. All points within the triangle shown in Fig. 9 are identified by $s = 0$, and all points outside this triangle are identified by $s = 1$. The numerical value of any bias weight should not exceed 1.5. How will the input data patterns $(0,0)$, $(2,0)$, $(4,-1)$, and $(-4,1)$ be classified?

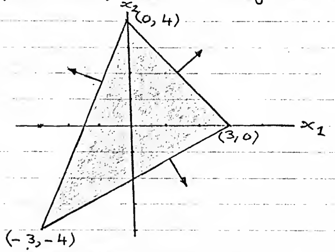



Fig. 9 Separation lines for Prob. 25

Solution

The first separation line [passing through points $(0,4)$ and $(-3,-4)$] is

$$\frac{x_2 - 4}{x_1 - 0} = \frac{-4 - 4}{-3 - 0}$$

or

$$3x_2 = 8x_1 + 12$$

## Problem 27

(a) State a mathematical formula for calculating the number of linear dichotomies that can be induced on $p$ points (in general position) in an $n$-dimensional space, $L(p,n)$.

(b) Use the formula of part (a) to show that

$$L(4,3) = L(4,2) + 2$$

(c) In view of the result of part (b), show that the logic XOR function can be implemented by the neural network of Fig. 10, where the space dimension increases from 2 to 3. Specify the role played by neuron 3, and determine the various weights of the network.

(d) In part (c), portray the separation plane in the 3-dimensional space. Comment on this situation from the dichotomization viewpoint.

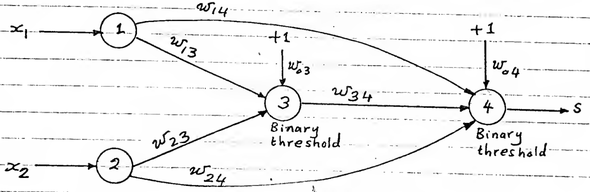



Fig. 10 Neural network for Prob. 27, part (c)

## Solution

(a) The mathematical formula for $L(p,n)$ is

26 Repeat Prob. 25 when the orientations of the separation lines are reversed.

28 In your solution of prob. 27, Part (c), specify the region of input patterns $(x_1, x_2)$ for which:

a) $X_3 = 0$

b) $X_3 = 1$

c) $X_3 = 0$ and $S = 0$

d) $X_3 = 0$ and $S = 1$

e) $X_3 = 1$ and $S = 0$

f) $X_3 = 1$ and $S = 1$

g) $S = 0$

h) $S = 1$

29 Repeat parts (c) and (d) of prob. 27 and Prob. 28 for the logic XNOR function.